UUCMS No. | | | | | | | | | | | | |

B.M.S. COLLEGE FOR WOMEN, AUTONOMOUS

BENGALURU – 560004

SEMESTER END EXAMINATION – SEPT/OCT 2023

M.Sc in Mathematics – 2nd Semester

PARTIAL DIFFERENTIAL EQUATIONS

Course Code: MM204T **QP Code: 12004**
Duration: 3 Hours **Max. Marks: 70**

Instructions:* *1) All questions carry equal marks.
2) Answer any five full questions.

1. a) Find the integral surface of the linear partial differential equation
$$x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z\,,$$
containing the straight line $x+y=0, z=1$.
b) Solve $yu_x + xu_y = u$ with condition $u(x,0)=x^3$, by the method of characteristics.

(7+7)

2. a) Solve the initial value problem $u_t + xu_x = x,\ 0 \le x \le 1, t > 0$ and $u(x,0)=2x$. Also show that the solution remains bounded as $t \to +\infty$.
b) Find the characteristics of the partial differential equation $pq = u$ and determine the integral surface which passes through the straight line $x=1, y=u$.

(7+7)

3. a) Classify the partial differential equation $u_{xx} + xu_{yy} = 0$ for $x \neq 0$.
In elliptic case, reduce it to its canonical form.
b) Classify the partial differential equation $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} = e^x$ and reduce it to canonical form.

(7+7)

4. a) Solve: $(D^3 - 2D^2D' - D\,D'^2 + 2D'^3)u = e^{x+y}$
b) Solve: $x^2r - y^2t = x^2y^3$
c) Solve: $(D^2 - 6DD' + 9D'^2)u = 12x^2 + 36xy$ **(5+4+5)**

5. a) Find the solution of wave equation in cylindrical polar form.
b) Obtain D'Alembert solution of IBVP: $u_{tt} = C^2\,u_{xx}\,,\ -\infty < x < \infty,\ t \ge 0$ with $u(x,0) = f(x),\ u_t(x,0) = g(x)\,,\ -\infty < x < \infty$. **(7+7)**

6. State and prove Dirichlet and Neumann problem involving Laplace equation in an upper half plane. **(14)**

7. a) Solve IBVP: $u_t = K\, u_{xx}$, $0 < x < \infty$, $t \geq 0$, subjected to $u(x,0) = e^{-x}$, $0 < x < 1, u_x(0,t) = 0, t > 0$, by applying appropriate Fourier transform.

   b) Solve IBVP: $u_t = K\, u_{xx}$, $0 \leq x \leq 1, t \geq 0$, subjected to $u(x,0) = x(1-x)$, $0 \leq x \leq 1, t \geq 0$, $u(0,t) = 0 = u(1,t)$, $t \geq 0$, using variable separable method. **(7+7)**

8. a) Solve IBVP: : $u_{tt} = K\, u_{xx}$, $0 \leq x \leq 1, t \geq 0$, subjected to $u(x,0) = f(x), \frac{\partial u}{\partial t}(x,0) = g(x)$, $0 \leq x \leq 1$ and $u(0,t) = 0 = u(1,t)$, $t \geq 0$ by using suitable integral transform method.

   b) Apply Duhamel's principle to obtain the solution of the non-homogeneous problem $u_t - u_{xx} = f(x), -\infty < x < \infty$, $t > 0$ with $u(x,0) = 0$, $-\infty < x < \infty$. **(7+7)**

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